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Technical Comment

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Comment on "Single-Cycle Unsteady Nozzle Phenomena in Pulse Detonation Engines"

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The authors of [1] state on page 329 that "For the nozzle designer looking for a simple, first-order method for identifying optimal expansion area ratio it would be particularly convenient to be able to identify $P_{o,\text{avg}}$ (the time-averaged head wall pressure) without having to perform CFD calculations." Fortunately, [2] offers one approach to estimating the time-averaged head pressure or, in our terminology, the equivalent exhaust total pressure $\tilde{p}_t$ and the optimal expansion area ratio. These quantities are easily and directly obtained for the either the ideal or the real pulse detonation engine (PDE) cycles of [2] by equating the exhaust velocity V_{10} obtained there [Eq. (8)] to the classical calorically perfect gas isentropic nozzle exhaust velocity V_e: viz.,

$$V_e^2 = V_0^2 + 2\eta_{\text{th}} f h_{\text{PR}} = 2C_p T_{t0}\left(1 + \frac{f h_{\text{PR}}}{C_p T_{t0}}\right)\left[1 - \left(\frac{p_0}{\tilde{p}_t}\right)^{\frac{\gamma-1}{\gamma}}\right] \quad (1)$$

where $\tilde{p}_t$ represents the equivalent exhaust total pressure, p_o is the freestream or ambient static pressure, V_0 is the freestream velocity, η_{th} is the thermal efficiency, f is the fuel-to-oxidizer ratio, h_{PR} is the lower heating value of the fuel, C_p is the specific heat at constant pressure, T_{t0} is the freestream total temperature, and γ is the ratio of specific heats. Note that the Nomenclature of [2] is used here and throughout this Technical Comment. Solving Eq. (1) for $\tilde{p}_t/p_0$ yields the desired result: namely,

$$\frac{\tilde{p}_t}{p_o} = \left[\frac{\psi + \tilde{q}}{\psi + \tilde{q} - \eta_{\text{th}}\tilde{q} - (\frac{\gamma-1}{2})M_0^2}\right]^{(\frac{\gamma}{\gamma-1})} > 1 \quad (2)$$

where M_0 is the freestream Mach number, $\tilde{q} = f h_{\text{PR}}/C_p T_0$ is the dimensionless heat supplied, and ψ is the static-temperature-rise ratio.

The corresponding expansion area ratio is then obtained from the traditional isentropic one-dimensional flow relationships:

$$\frac{A_e}{A^*} = \frac{1}{M_e}\left[\left(\frac{2}{\gamma+1}\right)\left(1 + \left\{\frac{\gamma-1}{2}\right\}M_e^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} \quad (3)$$

where A_e/A^* is the ratio of the exit area to the choked throat area, M_e is the exhaust Mach number, and

$$M_e^2 = \frac{2}{\gamma-1}\left[\left(\frac{\tilde{p}_t}{p_0}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \quad (4)$$

which, using Eq. (2), can be replaced by

$$M_e^2 = \left(\frac{2}{\gamma-1}\right)\left[\frac{(\frac{\gamma-1}{2})M_0^2 + \eta_{\text{th}}\tilde{q}}{\psi + \tilde{q} - \eta_{\text{th}}\tilde{q} - (\frac{\gamma-1}{2})M_0^2}\right] \quad (5)$$

to allow the impact of the input quantities to be directly visualized.

The methods of [2] are primarily used to determine the PDE cycle thermal efficiency η_{th}, which depends, in the ideal case, only upon $\tilde{q}$, γ, and the cycle static-temperature-rise ratio $\psi = T_3/T_0$ and, in the real case, also upon the compression efficiency η_c, the combustion efficiency η_b, and the expansion efficiency η_e. To simplify the ensuing analyses and make the results directly relevant to pure PDE propulsion, it is assumed in what follows that the cycle static-temperature-rise ratio is due entirely to the stagnation of the freestream flow [i.e., $\psi = T_3/T_0 = 1 + (\gamma-1)M_0^2/2$] or that the PDE cycle employs no additional mechanical compression, although the latter analysis is easily done with the methods of [2]. Further, the range of M_0 was chosen to bracket the ramjet regime to avoid the complications that lead to scramjets [3]. Figure 1 summarizes the results based upon this fundamental and transparent analysis for the stoichiometric C_2H_4/O_2 reaction ($f = 0.292$).

For the ideal case, which closely approaches the conditions of the computations of [1], Fig. 1 shows that A_e/A^* increases dramatically with M_0, primarily because of isentropic freestream compression. Designers should therefore be aware that A_e/A^* is quite sensitive to M_0 and that unusually high values of A_e/A^* may be required in flight

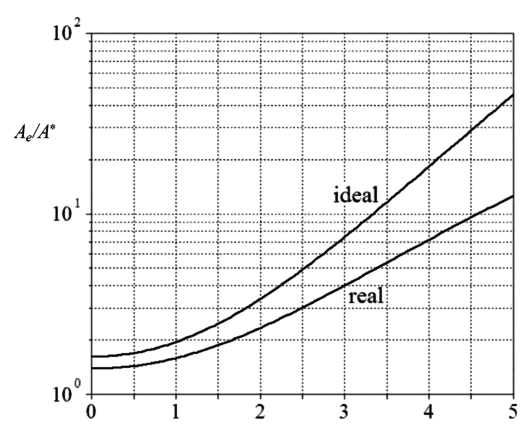

Fig. 1 Nozzle expansion area ratio A_e/A^* of the PDE as a function of flight Mach number M_0 for the stoichiometric C_2H_4/O_2 reaction (i.e., $\tilde{q} = 11$ and $\gamma = 1.24$). For the ideal case, $\eta_c = \eta_b = \eta_e = 1.0$ and $\eta_{\text{th}} = 0.3222$. For the real case, $\eta_c = \eta_b = \eta_e = 0.90$ and $\eta_{\text{th}} = 0.2803$.

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or simulated ground tests. For example, Eq. (5) reveals that M_e asymptotically approaches M_0 as the latter increases, and A_e/A^* will therefore increase accordingly. It should also be noted that the calculated value of A_e/A^* at $M_0 = 0$ (the static case) is 1.615, in close agreement with the value corresponding to the maximum I_{sp} for the computational results presented in Fig. 5 of [1]. This lends considerable confidence to the value of this approach as "a simple, first-order method for identifying optimal expansion area ratio" [1].

For the real case, Fig. 1 shows that A_e/A^* is also very sensitive to process efficiencies through their impact on η_{th} and that A_e/A^* increases less rapidly with M_0 than in the ideal case. This implies that any losses encountered in real PDE cycles must be carefully accounted for by the designer to arrive at suitable estimates of A_e/A^*.

References

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